VIDYA BHAWAN BALIKA VIDYA PITH

शक्तिउत्थानआश्रमलखीसरायबिहार

Class-12 Sub-.Maths Date 22.06..2021

5. Find an angle q, 0 < q < π/2, which increases twice as fast as its sine.

Solution:

According to the question, we have

$$\frac{d\theta}{dt} = 2\frac{d}{dt}(\sin\theta)$$

$$\Rightarrow \frac{d\theta}{dt} = 2\cos\theta \cdot \frac{d\theta}{dt} \quad \Rightarrow 1 = 2\cos\theta$$

$$\text{So, } \cos\theta = \frac{1}{2} \quad \Rightarrow \quad \cos\theta = \cos\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$$

Therefore, the required angle is π/3.

6. Find the approximate value of $(1.999)^5$.

Solution:

$(1.999)^5 = (2 – 0.001)^5$

Let x = 2 and Δx = -0.001

Also, let $y = x^5$

Differentiating both sides w.r.t, x, we get

$dy/dx = 5x^4 = 5(2)^4 = 80$

Now, Δy = (dy/dx). Δx = 80. (-0.001) = -0.080

And, $(1.999)^5 = y + \Delta y$

$= x^5 – 0.080 = (2)^5 – 0.080 = 32 – 0.080 = 31.92$

Therefore, approximate value of $(1.999)^5$ is 31.92

7. Find the approximate volume of metal in a hollow spherical shell whose internal and external radii are 3 cm and 3.0005 cm, respectively.

Solution:

Given,

The internal radius r = 3 cm

And, external radius R = r + Δr =3.0005 cm

Δr = 3.0005 – 3 = 0.0005 cm

Let $y = r^3 \Rightarrow y + \Delta y = (r + \Delta r)^3 = R^3 = (3.0005)^3$

Differentiating both sides w.r.t., r, we get

$$\frac{dy}{dr} = 3r^2$$

$$\text{So, } \Delta y = \frac{dy}{dr} \times \Delta r = 3r^2 \times 0.0005$$

$$= 3 \times (3)^2 \times 0.0005 = 27 \times 0.0005 = 0.0135$$

$$\therefore (3.0005)^3 = y + \Delta y \qquad \text{[From eq. } (i)\text{]}$$

$$= (3)^3 + 0.0135 = 27 + 0.0135 = 27.0135$$

$$\text{Volume of the shell} = \frac{4}{3}\pi[R^3 - r^3]$$

$$= \frac{4}{3}\pi[27.0135 - 27] = \frac{4}{3}\pi \times 0.0135$$

$$= 4\pi \times 0.005 = 4 \times 3.14 \times 0.0045 = 0.018\,\pi\,\text{cm}^3$$

Therefore, the approximate volume of the metal in the shell is 0.018π cm^3

$$1\frac{2}{3}$$

$$5\frac{1}{3}$$

8. A man, 2m tall, walks at the rate of m/s towards a street light which is m above

the ground. At what rate is the tip of his shadow moving? At what rate is the length of the

$$3\frac{1}{3}$$

shadow changing when he is m from the base of the light?

Solution:

Let AB is the height of street light post and CD is the height of the man such that

AB = 5(1/3) = 16/3 m and CD = 2 m

Let BC = x length (the distance of the man from the lamp post)

And CE = y is the length of the shadow of the man at any instant.

It's seen from the figure that,

Δ ABE ~ Δ DCE [by AAA similarity criterion]

Now, taking ratio of their corresponding sides, we have

$$\frac{AB}{CD} = \frac{BE}{CE} \Rightarrow \frac{AB}{CD} = \frac{BC + CE}{CE}$$

$$\frac{16/3}{2} = \frac{x+y}{y} \quad \Rightarrow \frac{8}{3} = \frac{x+y}{y}$$

$$8y = 3x + 3y \quad \Rightarrow 8y - 3y = 3x \quad \Rightarrow 5y = 3x$$

Differentiating both sides w.r.t, t, we have

$$5 \cdot \frac{dy}{dt} = 3 \cdot \frac{dx}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{3}{5} \cdot \frac{dx}{dt} \quad \Rightarrow \frac{dy}{dt} = \frac{3}{5} \cdot \left(-1\frac{2}{3}\right) = \frac{3}{5} \cdot \left(\frac{-5}{3}\right)$$

[∵ man is moving in opposite direction]

$$= -1 \text{ m/s}$$

So, the length of shadow is decreasing at the rate of 1 m/s.

Now, let u = x + y

(where, u = distance of the tip of shadow from the light post)

On differentiating both sides w.r.t. t, we get

$2\frac{2}{3}$

$$\frac{du}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$$

$$= \left(-1\frac{2}{3} - 1\right) = -\left(\frac{5}{3} + 1\right) = -\frac{8}{3} = -2\frac{2}{3} \text{ m/s}$$

Therefore, the tip of the shadow is moving at the rate of m/s towards the light post and the length of shadow decreasing at the rate of 1 m/s.